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Global Chaos Synchronization of Liu-Yang Systems via Sliding Mode Control

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Abstract— **This research work investigates the global chaos synchronization of identical chaotic systems based on sliding mode control theory and Lyapunov stability theory. The paper begins with a problem statement on global chaos synchronization of chaotic systems. A general result is derived for the global chaos synchronization of chaotic systems using slide mode control and this result is established using Lyapunov stability theory. Next, as an application of this general result, a sliding mode controller is derived for the global chaos synchronization of identical Liu-Yang chaotic systems (2010). Liu-Yang system is a new Lorenz-like chaotic system, which has very different fixed points: one saddle and two stable node-foci. Hence, this new chaotic system is topologically non-equivalent to Lorenz and other Lorenz-like systems. Moreover, in the sense defined by Vaněček and Čelikovský (1996), the Liu-Yang system (2010) connects the original Lorenz system (1964) and the original Chen system (1999) and represents a transition from one to the other. MATLAB simulations are shown to illustrate the sliding controller design for the global chaos synchronization of identical Liu-Yang chaotic systems.**

Keywords— **Chaos, chaotic systems, synchronization, sliding mode control, Lyapunov stability theory, Liu-Yang system.**

I. INTRODUCTION

Chaotic behaviour is an important feature, which is observed in some nonlinear dynamical systems. Chaotic behaviour was suspected well over 100 years ago, but it was established only a few decades ago due to the availability of computational power that enabled scientists to plot simulations of nonlinear dynamical systems.

A chaotic system is usually characterized by its extreme sensitivity of behaviour to initial conditions. Small changes in an initial state will make a very large difference in the behaviour of the system at the future states. This is usually called as the '*Butterfly Effect'*.

The Lyapunov exponent is a measure of the divergence of points that are initially very close and can be used to quantify chaotic systems. Thus, there is a spectrum of Lyapunov exponents, which are equal in number to the dimension of the phase space. It is common to refer to the largest Lyapunov exponent as the *maximal Lyapunov exponent* (MLE). A positive maximal Lyapunov exponent and phase space compactness are usually taken as defining conditions for a chaotic system.

In 1963, Lorenz discovered that a very small difference in the initial conditions led to large changes in his deterministic weather model [1]. There are many well-known paradigms of 3-dimensional chaotic systems like Rössler system ([2], 1976), Newton-Leipnik system ([3], 1981), Chen system ([4], 1999), Lü-Chen system ([5], 2002), Liu system ([6], 2004), Tigan system ([7], 2008), etc.

Chaotic systems have several applications in science engineering. Some important applications can be cited as secure communications [8-10], cryptosystems [11-12], physics [13-14], chemical reactions [15-16], biology [17-18], robotics [19-20], cardiology [21-23], neural networks [24], etc.

Synchronization of chaotic systems is a phenomenon that occurs when a chaotic system drives another chaotic system. Because of the butterfly effect in chaos theory, which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is apparently a very challenging problem in the chaos literature.

In most of the chaos synchronization problems, the master-slave or drive-response terminology is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of chaos synchronization is to use the output of the master system to control the slave system so that the states of the slave system track the states of the master system asymptotically.

In the last two decades, various schemes have been developed for the synchronization of chaotic systems such as PC method [25], OGY method [26], active control method [27-30], adaptive control method [31-34], backstepping control method [35-38], sampled-data feedback method [39], time-delay feedback method [40], sliding mode control method [41-45], etc.

In this paper, we derive a general result for the global chaos synchronization of chaotic systems using sliding mode control (SMC) theory [46-48].

The sliding mode control approach is recognized as an efficient tool for designing robust controllers for linear or nonlinear control systems operating under uncertainty conditions.

A major advantage of sliding mode control is low sensitivity to parameter variations in the plant and disturbances affecting the plant, which eliminates the necessity of exact modeling of the plant.

In the sliding mode control theory, the control dynamics will have two sequential modes, viz. the *reaching mode* and the *sliding mode*. Basically, a sliding mode controller (SMC) design consists of two parts: hyperplane design and controller design. A hyperplane is first designed via the pole-placement approach in the modern control theory and a controller is then designed based on the sliding condition. The stability of the overall system is guaranteed by the sliding condition and by a stable hyperplane.

This research work is organized as follows. Section II discusses the problem statement and research methodology. In this section, we detail the sliding mode controller (SMC) design for the global chaos synchronization of identical chaotic systems.

The rest of the paper deals with the sliding mode controller design for the global chaos synchronization of identical Liu-Yang systems ([49], 2010).

Section III describes the qualitative properties of the strange attractor exhibited by the Liu-Yang system.

In Section IV, we describe the sliding mode controller design for the global chaos synchronization of identical Liu-Yang systems. MATLAB simulations are shown to validate and illustrate the sliding mode controller design for the global chaos synchronization for the Liu-Yang systems. Section V contains the conclusions of this research work.

II. Main Results

First, we give a problem statement of global chaos synchronization of a pair of chaotic systems called the *master* and *slave systems.*

As the *master system*, we consider the chaotic system

$$\dot{x} = Ax + f(x), \tag{1}$$

where $x \in R^n$ is the state of the system, A is the matrix of system parameters and $f(x)$ contains the nonlinear parts of the system.

As the *slave system*, we consider the controlled (identical) chaotic system

$$\dot{y} = Ay + f(y) + u, \tag{2}$$

where $y \in R^n$ is the state of the system, and u is the controller to be designed.

The *synchronization error* is defined by

$$e = y - x \tag{3}$$

Then the error dynamics is obtained as

$$\dot{e} = Ae + f(y) - f(x) \tag{4}$$

which can be equivalently expressed as

$$\dot{e} = Ae + g(x,y) + u, \tag{5}$$

where

$$g(x,y) = f(y) - f(x). \tag{6}$$

For the SMC design, we first set

$$u(t) = -g(x,y) + Bv(t), \tag{7}$$

where B is an $(n \times 1)$ column vector chosen such that (A, B) is controllable.

If we substitute (7) into (5), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae + Bv, \tag{8}$$

which is a linear time-invariant control system with single input v.

Hence, we have converted the original problem of global chaos synchronization of identical chaotic systems (1) and (2) into an equivalent problem of globally stabilizing the error dynamics (8) by a suitable choice of the feedback control (7).

In the SMC design, we first define the sliding variable as

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n, \tag{9}$$

where C is an $(1 \times n)$ row vector to be determined.

The *sliding manifold* S is defined as the hyperplane

$$S = \left\{ e \in R^n : s(e) = Ce = 0 \right\} \tag{10}$$

where C is chosen so that (C, A) is controllable.

Let us assume that a sliding motion occurs on S.

In sliding mode,

$$s \equiv 0 \quad \text{and} \quad \dot{s} = CAe + CBv = 0 \tag{11}$$

Assuming that $CB \neq 0$, the sliding motion is affected by the so-called equivalent control given by

$$v_{\text{eq}}(t) = -(CB)^{-1} CA\, e(t) \tag{12}$$

Consequently, the equivalent dynamics in the sliding phase is defined by

$$\dot{e} = [I - B(CB)^{-1}C]Ae = Ee, \tag{13}$$

where

$$E = [I - B(CB)^{-1}C]A \tag{14}$$

It is easy to very that E is independent of the control and has at most $(n-1)$ nonzero eigenvalues, depending on the chosen switching surface, while the associated eigenvectors belong to $\ker(C)$.

Since (C, A) is controllable, by the procedure given in sliding control theory [46-48], we can choose C so that E has any desired $(n-1)$ stable eigenvalues.

Thus, the dynamics in the sliding mode is globally asymptotically stable.

Finally, for the SMC design, we use the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - ks \tag{15}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0$, $k > 0$ are found so that the sliding condition is satisfied and the sliding motion will occur.

From the equations (11) and (12), we finally get the sliding control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s) \right] \tag{16}$$

The main result of this section is stated and proved as follows.

Theorem 1. The identical chaotic systems (1) and (2) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in R^n$ by the sliding control law

$$u(t) = -g(x, y) + Bv(t) \qquad (17)$$

where v is as defined by Eq. (16), B is an $(n \times 1)$ vector such that (A, B) is controllable, C is an $(1 \times n)$ vector such that (C, A) is controllable and that the matrix E defined by (14) has $(n-1)$ stable eigenvalues.

Proof. Substituting the control laws (17) and (16) into the error dynamics (5), we obtain

$$\dot{e} = Ae - B(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \qquad (18)$$

We prove the global asymptotic stability of the error system (18) by considering the candidate Lyapunov function

$$V(e) = \frac{1}{2}s^2(e) \qquad (19)$$

The sliding mode motion is characterized by the equations

$$s(e) = 0 \text{ and } \dot{s}(e) = 0 \qquad (20)$$

By the choice of E, the dynamics in the sliding mode is globally asymptotically stable.

When $s(e) \neq 0, V(e) > 0.$

Also, when $s(e) \neq 0,$ differentiating V along the error dynamics (18) or the equivalent dynamics (15), we obtain

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\text{sgn}(s) < 0 \qquad (21)$$

Hence, by Lyapunov stability theory [50], the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in R^n$.

This completes the proof. ■

III. Strange Attractor of Liu-Yang System

This section gives details of the strange attractor of Liu-Yang system ([49], 2010), which is a new Lorenz-like chaotic system.

The Liu-Yang system is described by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_1 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \qquad (22)$$

where x_1, x_2, x_3 are the states and a, b, c are constant, positive parameters of the system.

The Liu-Yang system (22) has a strange attractor for the parametric values

$$a = 35, \quad b = 3, \quad c = 35 \qquad (23)$$

The strange attractor of the Liu-Yang system (22) is shown in Fig. 1. Also, the Lyapunov exponents are found as

$$\lambda_{LE1} = 0.57018, \quad \lambda_{LE2} = 0, \quad \lambda_{LE3} = -13.237$$

The Liu-Yang system (22) has one saddle and two stable node-foci. Thus, the Liu-Yang system (22) is topologically non-equivalent to the original Lorenz and other Lorenz-like chaotic systems.

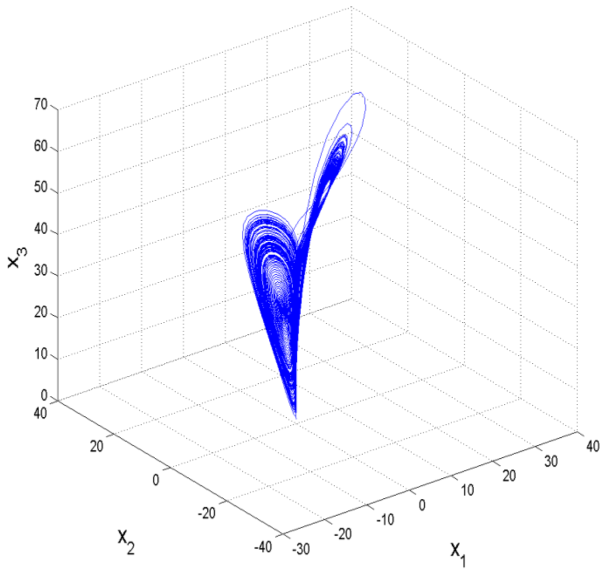

Fig. 1 Strange Attractor of the Liu-Yang System

IV. Sliding Mode Controller Design for the Global Chaos Synchronization of Liu-Yang Systems

In this section, we use the results of Section II to derive a new sliding mode controller for achieving global chaos synchronization of identical Liu-Yang systems (2010).

As the master system, we take the Liu-Yang system

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= cx_1 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \qquad (24)$$

where x_1, x_2, x_3 are the states and a, b, c are constant, positive parameters of the system.

As the slave system, we take the controlled Liu-Yang system

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= cy_1 - y_1y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3 \end{aligned} \qquad (25)$$

where y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the controllers to be determined.

The synchronization error is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \end{aligned} \qquad (26)$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= ce_1 - y_1y_3 + x_1x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1y_2 - x_1x_2 + u_3 \end{aligned} \qquad (27)$$

We can rewrite the error dynamics (27) in matrix form as

$$\dot{e} = Ae + g(x, y) + u \tag{28}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ c & 0 & 0 \\ 0 & 0 & -b \end{bmatrix} \tag{29}$$

and

$$g(x, y) = \begin{bmatrix} 0 \\ -y_1 y_3 + x_1 x_3 \\ y_1 y_2 - x_1 x_2 \end{bmatrix} \tag{30}$$

The sliding mode controller u for achieving global chaos synchronization of the Liu-Yang systems (24) and (25) is carried out by the procedure outlined in Section II.

First, we set u as

$$u(t) = -g(x, y) + Bv(t), \tag{31}$$

where B is chosen so that (A, B) is controllable.

A simple choice of B is

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{32}$$

In the chaotic case, the parameter values of the Liu-Yang system are chosen as

$$a = 35, \quad b = 3, \quad c = 35 \tag{33}$$

The sliding mode variable is chosen as

$$s = Ce = \begin{bmatrix} 6 & -1 & 0 \end{bmatrix} e = 6e_1 - e_2 \tag{34}$$

which renders the dynamics in sliding mode asymptotically stable.

Next, we take the sliding mode gains as

$$k = 6 \text{ and } q = 0.2 \tag{35}$$

From Eq. (16) of Section II, we obtain the control v as

$$v(t) = 41.8e_1 - 40.8e_2 - 0.04\,\text{sgn}(s) \tag{36}$$

Applying Theorem 1, we get the following result.

Theorem 2. The identical Liu-Yang systems (24) and (25) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in R^3$ with the sliding mode controller u defined by (31). ■

For numerical simulations, we use classical fourth-order Runge-Kutta method (MATLAB) with step-size $h = 10^{-8}$ for solving the identical Liu-Yang systems (24) and (25) when the active sliding mode control u defined by (31) is applied.

The initial conditions of the Liu-Yang system (24) are taken as

$$x_1(0) = 3.4, \; x_2(0) = -5.7, \; x_3(0) = 6.1$$

The initial conditions of the Liu-Yang system (24) are taken as

$$y_1(0) = 7.2, \; y_2(0) = 3.5, \; y_3(0) = -4.8$$

Fig.2 depicts the synchronization of the identical Liu-Yang chaotic systems (24) and (25).

Fig. 3 depicts the time-history of the synchronization errors e_1, e_2, e_3.

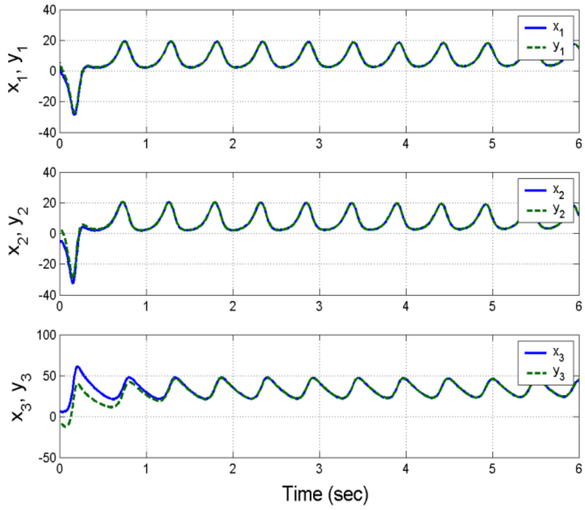

Fig. 2 Synchronization of Identical Liu-Yang Chaotic Systems

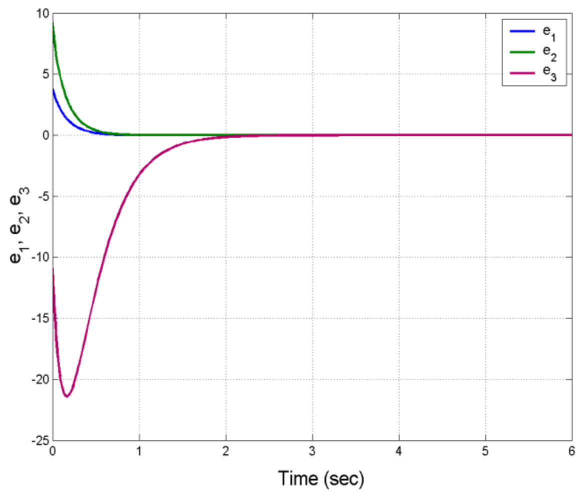

Fig. 3 Time-History of the Synchronization Errors e_1, e_2, e_3

V. Conclusions

Synchronization of chaotic systems is an important research problem in chaos literature where two chaotic systems, called *master* and *slave* systems, are synchronized using control laws. This paper derived new results for the design of sliding mode controllers for identical chaotic systems. The main result was proved using Lyapunov stability theory. As an application of this result, an effective sliding mode controller was designed for the global chaos synchronization of identical Liu-Yang systems (2010). Numerical simulations using MATLAB were shown to validate and demonstrate our sliding mode controller design for the global chaos synchronization of identical Liu-Yang systems.

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